Name $\qquad$ Date $\qquad$
Discrete Math Midterm Review 2010
The Midterm exam is all multiple choice scantron. Be sure to bring a calculator and a number 2 pencil. You may use a notecard that must be approved ahead of time. NOTA= none of the above.

1. Which is a circuit that traverses each edge of the graph exactly once?
A. Euler circuit.
B. Hamilton Circuit
C. Minimum Spanning Tree
D. any circuit
2. Which is a circuit that traverses each vertex of the graph exactly once?
A. Euler circuit.
B. Hamilton Circuit
C. Minimum Spanning Tree
D. any circuit
3. Which of the following is a valid way to Eulerize a graph?
A. Adding vertices and edges so that the graph can be traversed without backtracking.
B. Eliminating edges that cannot be reached without backtracking.
C. Adding duplicate edges so that a circuit exists that traverses each edge of the graph once.
D. Making sure that there is an even number of odd vertices when you are done.

Use the following graph to answer questions \# 4-12.

4. Is the graph connected?
A. Yes
B. No
C. Cannot be determined.
5. How many edges are on the graph?
A. 8
B. 9
C. 10
D. 11
E. NOTA
6. How many vertices are on the graph?
A. 8
B. 9
C. 10
D. 11
E. NOTA
7. What is the valence of vertex $L$ ?
A. 0
B. 1
C. 2
D. 3
E. NOTA

8. What is the valence of vertex U ?
A. 1
B. 2
C. 3
D. 4
E. NOTA
9. Does the graph have an Euler Path?
A. Yes
B. No
C. Cannot be determined.
10. What is the best description for $\mathrm{L}, \mathrm{U}, \mathrm{T}, \mathrm{A}, \mathrm{H}, \mathrm{M}, \mathrm{I}$ ?
A. path
B. circuit
C. Euler path
D. Euler circuit
11. What is the best description for $\mathrm{L}, \mathrm{U}, \mathrm{T}, \mathrm{A}, \mathrm{H}, \mathrm{M}, \mathrm{I}, \mathrm{L}$ ?
A. path
B. circuit
C. Euler path
D. Euler circuit
12. What is the best description for $\mathrm{V}, \mathrm{U}, \mathrm{T}, \mathrm{A}, \mathrm{U}, \mathrm{L}, \mathrm{A}, \mathrm{H}, \mathrm{M}, \mathrm{I}, \mathrm{L}$ ?
A. path
B. circuit
C. Euler path
D. Euler circuit
13. If a graph is connected and $\qquad$ , the graph will have an Euler Circuit.
a. the graph has an even number of vertices
b. the graph has an even number of edges
c. the graph has all vertices of even valence
d. the graph has only two odd vertices
14. Is this statement always, sometimes or never true?

A graph that is not connected must have at least one vertex with valence 0 .
A. Always True
B. Sometimes True
C. Never True
15. Is this statement always, sometimes or never true?

Every connected graph has an Euler circuit.
A. Always True
B. Sometimes True
C. Never True
16. Consider the path represented by the numbered sequence of edges on the graph below.


Which statement is true?
A. The path is not a circuit.
B. The path is an Euler Circuit.
C. The path is a circuit, but not an Euler circuit.
D. None of the above.
17. Which of the following statements about a connected graph is always true?
A. Every pair of vertices is joined by a single edge.
B. A path of edges exists between any two vertices of the graph.
C. There is an even number of vertices on the graph.
D. There is an even number of edges on the graph.
18. Which of the graphs below are connected?

A. I only.
B. II only.
C. Both I and II.
D. Neither I nor II.
19. Consider the paths represented by the numbered sequence of edges on the graph below. Which path represents an Euler circuit?
I.

II.

A. I only.
B. II only.
C. Both I and II.
D. Neither I nor II.
20. What is the valence of vertex A in the graph below?

A. 4
B. 5
C. 7
D. 11
21. Which of the graphs below have Euler circuits?

A. I only.
B. II only.
C. Both I and II.
D. Neither I nor II.
22. After a major natural disaster, such as a flood, hurricane, or tornado, many tasks need to be completed as efficiently as possible. For which situation below would finding an Euler circuit or an efficient Eulerization of a graph be the appropriate mathematical technique to apply?
A. The department of Public Works must inspect all streets in the city to remove dangerous debris.
B. Relief food supplies must be delivered to eight emergency shelters located at different sites in a large city.
C. The Department of Public Works must inspect traffic lights at intersections in the city to determine which are still working.
D. An insurance claims adjuster must visit 10 homes in various neighborhoods to write reports.
23. For which of the two situations is it desirable to find an Euler circuit or an efficient eulerization of a graph?
I. Plowing the streets of a small village after a snow.
II. Painting lines down the center of all the roads in a town.
A. I only.
B. II only.
C. Both I and II.
D. Neither I nor II.
24. Which of the following graphs shown below give the best Eulerization of the given graph?

a.

c.

b.

d.

25. Which of the following graphs shown below give the best Eulerization of the given graph?
a.

b.

c.

d.


26. In order to eulerize the graph below, give the fewest number of edges that need to be added or duplicated.

A. 1
B. 2
C. 3
D. 4
27. The map below shows the territory for a parking control officer. The dots represent parking meters that need to be checked. Which graph would be useful for finding an efficient route?
(That is, which graph represents the situation as it is on the map below.)

a.

b.

c.

d.

28. Which of the following graphs has a bridge (an edge that when removed will disconnect the graph)?
A.

B.

C.
D. NOTA
29. Which of the following graphs is not possible?
A. a graph that has all vertices with even valance, but does not have an Euler circuit
B. a graph that has an odd number of vertices and an Euler circuit
C. a graph with a vertex of valence 0
D. a graph with 2 odd vertices that cannot be Eulerized with one duplicate edge
E. a graph with five vertices of valence $1,1,2,2$, and 3 .
30. Which square will Bob end up in?

## MAIN:



31. Suppose a pizza delivery person needs to take pizzas to 10 houses in different neighborhoods and then return to pick up the next set to be delivered. The technique most likely to be used in solving this problem is
A) finding an Euler circuit on a graph.
B) applying the nearest-neighbor algorithm for the traveling salesman problem.
C) applying Kruskal's algorithm for finding a minimum-cost spanning tree for a graph.
D) None of these techniques is likely to apply.
32. Suppose an employee of a power company needs to read the electricity meters outside of each house along the streets in a residential area. The technique most likely to be useful in solving this problem is
A) finding an Euler circuit on a graph.
B) applying the nearest-neighbor algorithm for the traveling salesman problem.
C) applying Kruskal's algorithm for finding a minimum-cost spanning tree for a graph.
D) None of these techniques is likely to apply.
33. A college student has six pairs of pants, eight tee shirts, three sweatshirts and two pairs of tennis shoes. If an outfit consists of pants, a tee shirt, a sweatshirt, and a pair of tennis shoes, how many different outfits can the student wear before repeating one?
A) 19
B) 124
C) 288
D) 328
E) NOTA

For problems 34-37. Highlight the route of the graph, and then give the solution.
34. Find the Hamilton circuit obtained by using the sorted-edges algorithm (cheapest link.)

A) ACBEDA
B) ABCEDA
C) ABEDCA
D) ADCEBA
E) NOTA
35. Find the cost of the circuit obtained by using the sorted-edges algorithm (cheapest link.)

A) 40
B) 58
C) 60
D) 66
E) NOTA
36. Find the Hamilton circuit obtained by using the Nearest-Neighbor Algorithm starting at vertex A.

A) ABCEDA
B) ABEDCA
C) ADCEBA
D) ABCED
E) NOTA
37. Find the cost of the circuit obtained by using the Nearest-Neighbor Algorithm starting at vertex A.

A) 215
B) 220
C) 235
D) 295
E) NOTA

Consider the preference schedule in an election with 5 candidates for questions 38-49.

|  | 7 | 5 | 4 | 3 | 2 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1st | $A$ | $B$ | $C$ | $D$ | $E$ |
| 2nd | $B$ | $D$ | $D$ | $B$ | $D$ |
| 3rd | $C$ | $C$ | $E$ | $A$ | $A$ |
| 4 th | $D$ | $E$ | $B$ | $C$ | $B$ |
| Stn | $E$ | $A$ | $A$ | $E$ | $C$ |

38. How many voters voted in this election?
A. 5
B. 6
C. 20
D. 21
E. NOTA
39. How many votes are needed for a majority (more than $50 \%$ of the vote)?
A. 10
B. 11
C. 50
D. 51
E. NOTA
40. How many first place votes does candidate A have?
A. 5
B. 6
C. 7
D. 4
E. NOTA
41. How many first place votes does candidate $B$ have?
A. 5
B. 6
C. 7
D. 4
E. NOTA
42. How many first place votes does candidate C have?
A. 5
B. 6
C. 7
D. 4
E. NOTA
43. How many first place votes does candidate D have?
A. 0
B. 1
C. 2
D. 3
E. NOTA
44. How many first place votes does candidate E have?
A. 0
B. 1
C. 2
D. 3
E. NOTA
45. Who is the winner of the election by the Plurality Method?
A. A
B. B
C. C
D. D
E. E
46. Who is the first candidate to be eliminated in the Plurality with Elimination (Hare) method?
A. A
B. B
C. C
D. D
E. E
47. When the candidate in \#46 gets eliminated, which candidate gets those votes in the second round?
A. A
B. B
C. C
D. D
E. E
48. Which candidate gets eliminated in the second round?
A. A
B. B
C. C
D. D
E. E
49. In the Plurality with Elimination (Hare) method, who wins?
A. A
B. B
C. C
D. D
E. E

Consider the following preference schedule in an election with 3 candidates for questions 50-59

|  | 1 | 1 | 1 | 1 | 1 | 2 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1st | A | A | B | B | C | C |
| 2nd | B | C | A | C | A | B |
| 3rd | C | B | C | A | B | A |

50. In the Borda Count method, how many points does each candidate get for a first place vote?
A. 0
B. 1
C. 2
D. 3
E. NOTA
51. In the Borda Count method, how many points does each candidate get for a 2 nd place vote?
A. 0
B. 1
C. 2
D. 3
E. NOTA
52. What is candidate A's Borda score?
A. 6
B. 7
C. 8
D. 9
E. NOTA
53. What is candidate B's Borda score?
A. 6
B. 7
C. 8
D. 9
E. NOTA
54. What is candidate C's Borda score?
A. 6
B. 7
C. 8
D. 9
E. NOTA
55. Who is the winner by the Borda Count method?
A. A
B. B
C. C
D. NOTA (there is a tie)
56. In the method of Pairwise Comparisons, who wins in the $\mathrm{A} v \mathrm{~B}$ ?
A. A
B. B
C. C
D. NOTA (there is a tie)
57. In the method of Pairwise Comparisons, who wins in the $\mathrm{A} v \mathrm{C}$ ?
A. A
B. B
C. C
D. NOTA (there is a tie)
58. In the method of Pairwise Comparisons, who wins in the B v C ?
A. A
B. B
C. C
D. NOTA (there is a tie)
59. Who is the winner in the method of Pairwise Comparisons?
A. A
B. B
C. C
D. NOTA (there is a tie)
60. 

How many votes are needed for a majonity winner if there are 20 voters?
A) 10
B) 11
C) 15
D) 20

Consider the preference schedule in an election with 5 candidates for questions 61-65.

|  | 7 | 5 | 4 | 3 | 2 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1st | $A$ | $B$ | $C$ | $D$ | $E$ |
| 2nd | $B$ | $D$ | $D$ | $A$ | $D$ |
| 3rd | C | $C$ | $E$ | $B$ | $A$ |
| 4n | $D$ | $E$ | $A$ | $C$ | $B$ |
| 5nn | $E$ | $A$ | $B$ | $E$ | $C$ |

61. In the method of sequential pairwise voting with the agenda of $\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D}, \mathrm{E}$, who wins in AvB ?
A. A
B. B
C. C
D. D
E. E
F. NOTA (there is a tie)
62. In the method of sequential pairwise voting with the agenda of $A, B, C, D, E$, who will the winner of AvB be paired with in the second match-up?
A. A
B. B
C. C
D. D
E. E
63. In the method of sequential pairwise voting with the agenda of $A, B, C, D, E$, who wins round 2 ?
A. A
B. B
C. C
D. D
E. E
F. NOTA (there is a tie)
64. In the method of sequential pairwise voting with the agenda of $\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D}, \mathrm{E}$, who wins round 3 ?
A. A
B. B
C. C
D. D
E. E
F. NOTA (there is a tie)
65. In the method of sequential pairwise voting with the agenda of $\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D}, \mathrm{E}$, who wins the election?
A. A
B. B
C. C
D. D
E. E
F. NOTA (there is a tie)

For questions 66-70, match each criterion to the letter choice that describes it.
66. $\qquad$ Majority
67. $\qquad$ Condorcet
68. $\qquad$ Monotonicity
69. $\qquad$ Independence-of-Irrelevant-Alternatives
70. $\qquad$ Arrow's Impossibility Thereom
A. If choice $X$ is a winner of an election, and in a reelection, the only changes in the ballots are changes that favor X , then X should remain a winner of the election.
B. If there is a choice that has a majority of the first-place votes in an election, then that choice should be the winner of the election
C. If a candidate X is a winner of an election and one (or more) of the other candidates is removed and the ballots recounted, then X should still be the winner of the election.
D. The discovery that any voting system for more than 2 candidates can give undesirable outcomes and will be unfair.
E. If there is a choice that in a head-to-head comparison is preferred by the voters over every other choice, then that choice should be the winner of the election.
71. Consider the following election with 3 candidates:

|  | 1 | 1 | 1 |
| :---: | :---: | :---: | :---: |
| 1st | $A$ | $B$ | $C$ |
| 2nd | $B$ | $C$ | $A$ |
| 3rd | C | $A$ | $B$ |

Which agenda yields A the winner?
A. A,B,C
B. B,C,A
C. $\mathrm{A}, \mathrm{C}, \mathrm{B}$
D. A cannot win with any agenda
72. Consider the following preference schedule for an election with 4 candidates for questions 72-74.

Who is the Condorcet Candidate?
A. A
B. B
C. C
D. D
E. NOTA
73. Who is the winner of the election using the Borda Count method.
A. A
B. B
C. C
D. D
E. NOTA
74. What criterion does the Borda Count method violate based on question 72 and 73 ?
A. Majority
B. Condorcet
C. Monotonicity
D. Independence-of-Irrelevant-Alternatives
E. NOTA
75. Consider each game below of rock-paper-scissors-dynamite-water. What should you select to get the best outcome for you? (You prefer a win to a tie or loss and prefer a tie to a loss.)

| you | $X$ | $Y$ |
| :---: | :---: | :---: |
|  | $P$ | $D$ |

A. R
B. S
C. P
D. W
E. D
76. Consider each game below of rock-paper-scissors-dynamite-water. What should you select to get the best outcome for you? (You prefer a win to a tie or loss and prefer a tie to a loss.)

| you | X | Y |
| :---: | :---: | :---: |
|  | S | $D$ |

A. R
B. S
C. P
D. W
E. D

Rock splashes water and breaks scissors.
Scissors cuts paper and dynamite.
Dynamite blows up rock and burns paper.

Paper covers rock and soaks up water.
Water rusts scissors and puts out dynamite.
77. Consider the voting system: [ $\mathrm{q}, 3,2,1$ ].
a. What would the quota be if a simple majority is required to pass a motion?
A. 3
B. 4
C. 5
D. 6
E. NOTA
78. What would the quota be if unanimous vote is required to pass a motion?
A. 3
B. 4
C. 5
D. 6
E. NOTA
79. What could NOT be the quota, q , in the voting system: $[\mathrm{q}, 4,3,2,1]$ ?
A. 5
B. 6
C. 7
D. 8
E. 9
80. In the weighted voting system, $[9: 10,5,3], \mathrm{P} 1$ is a $\qquad$ .
A. dictator
B. dummy
C. player with veto power
D. NOTA
81. In the weighted voting system, [ $38: 20,15,12,5], \mathrm{P} 2$ is a $\qquad$ .
A. dictator
B. dummy
C. player with veto power
D. NOTA
82. In the weighted voting system, $[9: 6,4,2], \mathrm{P} 3$ is a $\qquad$ .
A. dictator
B. dummy
C. player with veto power
D. NOTA
83. Consider the following voting system: [ $14: 8,7,5,4,2$ ]. Who is the pivotal player in the sequential coalition $<8,7,5,4,2>$ ? (Shapley-Shubik)
A. $\mathrm{Pl}=8$
B. $\mathrm{P} 2=7$
C. $\mathrm{P} 3=5$
D. $\mathrm{P} 4=4$
E. $P 5=2$
84. Consider the following voting system: [ $14: 8,7,5,4,2$ ]. Which players are critical in the winning coalition $\{7,5,4,2\}$ ? (Banzhaf)
A. $\mathrm{P} 1=8$ and $\mathrm{P} 2=7$
B. $\mathrm{P} 2=7$ and $\mathrm{P} 3=5$
C. $\mathrm{P} 1=8, \mathrm{P} 2=7$, and $\mathrm{P} 3=5$
D. $P 2=7$
E. $\mathrm{P} 4=4$ and $\mathrm{P} 5=2$
85. How many sequential coalitions are there in the weighted voting system [ $14: 8,7,5,4,2$ ]? (Shapley-Shubik)
A. 24
B. 120
C. 32
D. 64
E. NOTA
86. How many different coalitions (winning and losing) are there in the weighted voting system [ $14: 8,7,5,4,2$ ]? (Banzhaf)
A. 24
B. 120
C. 32
D. 64
E. NOTA
87. The best description for a voter who always wins is $\qquad$ .
A. dictator
B. dummy
C. player with veto power
D. lucky
88. Which is a voter in a coalition whose defection changes the coalition from a winning coalition to a losing coalition?
A. pivotal
B. critical
C. not loyal
D. loser
89. What is the weight of a voter in a weighted voting system?
A. the number of pounds the voter is
B. the number of votes assigned to the voter
C. the number of times the voter is critical
D. the number of times the voter is pivotal
90. A voter that is never pivotal in any sequential coalition is $\qquad$ .
A. dictator
B. dummy
C. player with veto power
D. NOTA
91. Evaluate $C_{5}^{7}$.
A. 75
B. 15
C. 21
D. 14
E. NOTA
92. What is middle number in the fifth row of Pascal's Triangle?
A. 3
B. 4
C. 5
D. 6
E. NOTA
93. What is the Shapley-Shubik power index as a percent of P 1 in the weighted voting system, [ $4: 3,2,1$ ] given all the sequential coalitions with pivotal players circled below?

$$
\langle 3,2,1\rangle\langle 3, \text { (1) } 2\rangle\langle 2,3,1\rangle\langle 2,1,3\rangle\langle 1,2,3\rangle\langle 1,3,2\rangle
$$

A. $50 \%$
B. $60 \%$
C. $70 \%$
D. $80 \%$
E. NOTA
94. What is the Banzhaf power index as a percent of P 1 in the weighted voting system, $[4: 3,2,1]$ given all the winning coalitions with critical players circled below? $\{(3,2,1\} \quad\{3,(2)\} \quad\{3,11\}$
A. $3 \%$
B. $30 \%$
C. $40 \%$
D. $60 \%$
E. NOTA

For questions 95-97, find the Banzhaf power index as a percent for each player in the weighted voting system: [6:5, 3, 3].
95. P1's power index is $\qquad$ .
A. $33 \frac{1}{3} \%$
B. $66 \frac{2}{3} \%$
C. $40 \%$
D. $60 \%$
E. NOTA
96. P 2 's power index is $\qquad$ .
A. $33 \frac{1}{3} \%$
B. $66 \frac{2}{3} \%$
C. $40 \%$
D. $60 \%$
E. NOTA
97. P3's power index is $\qquad$ .
A. $33 \frac{1}{3} \%$
B. $66 \frac{2}{3} \%$
C. $40 \%$
D. $60 \%$
E. NOTA

For questions 98-100, find the Shapley-Shubik power index as a percent for each player in the weighted voting system: [6:5, 3, 2 ].
98. P1's power index is $\qquad$ .
A. $33 \frac{1}{3} \%$
B. $66 \frac{2}{3} \%$
C. $50 \%$
D. $0 \%$
E. NOTA
99. P 2 's power index is $\qquad$ .
A. $33 \frac{1}{3} \%$
B. $66 \frac{2}{3} \%$
C. $50 \%$
D. $0 \%$
E. NOTA
100. P3's power index is $\qquad$ .
A. $33 \frac{1}{3} \%$
B. $66 \frac{2}{3} \%$
C. $50 \%$
D. $0 \%$
E. NOTA
101. Sam and Adam are dividing items using the Adjusted Winner Procedure.

If Sam gets $3 / 13$ of the Skittles, what fraction of the Skittles does Adam get?
A. $\frac{3}{13}$
B. $\frac{10}{13}$
C. $\frac{9}{13}$
D. $\frac{1}{13}$
E. NOTA
102. Sam and Adam are dividing items using the Adjusted Winner Procedure.

If Sam gets the fraction x of the Skittles, what fraction of the Skittles does Adam get?
A. x
B. $x-1$
C. $1-\mathrm{x}$
D. $1+\mathrm{x}$
E. NOTA
103. Solve for $\mathrm{x}: 35+15 \mathrm{x}=40+20(1-\mathrm{x}) . \quad \mathrm{x}=$ $\qquad$ .
A. $\frac{5}{7}$
B. $\frac{2}{7}$
C. $\frac{7}{19}$
D. $\frac{1}{19}$
E. NOTA
104. Chinah is bids $\$ 1000$ for a painting in a the Knaster Inheritance Procedure. What is her fair share if the painting is being divided by 4 people?
A. $\$ 1000$
B. $\$ 500$
C. $\$ 400$
D. $\$ 250$
E. NOTA

Consider the following problem for questions 105-. 108
Andrew and Keith's parents are selling their cape house. They told Keith and Andrew that they could split the items left in the house. Keith and Andrew have decided to use the Adjusted Winner Procedure and have assigned points to the items as follows:

|  | Andrew | Keith |
| :--- | :--- | :--- |
| Kayak | 10 | 15 |
| sail boat | 20 | 15 |
| wake board | 10 | 10 |
| Wii video game | 15 | 20 |
| HD TV | 25 | 30 |
| Stereo | 10 | 5 |
| Computer | 10 | 5 |

105. How do you determine who gets the items initially?
A. highest bidder
B. lowest bidder
C. coin toss
D. NOTA
106. Who gets the tie (wake board ) initially?
A. Andrew
B. Keith
107. What item is shared?
A. Kayak
B. Wii Video game
C. HD TV
D. wake board
108. What fraction of the shared item does Andrew get?
A. $\frac{3}{11}$
B. $\frac{8}{11}$
C. $\frac{7}{19}$
D. $\frac{12}{19}$
E. NOTA
109. Five players want to divide a cake fairly using the lone divider method. The divider cuts the cake into 5 slices. Determine a possible fair divisions of the cake given the following bids:
a. $\mathrm{C} 1=\{\mathrm{s} 2, \mathrm{~s} 3\}$
$\mathrm{C} 2=\{\mathrm{s} 3\}$
$\mathrm{C} 3=\{\mathrm{s} 1, \mathrm{~s} 4\}$
$\mathrm{C} 4=\{\mathrm{s} 1\}$
A. $\mathrm{D}=\{\mathrm{s} 5\}$
$\mathrm{C} 1=\{\mathrm{s} 3\}$
$\mathrm{C} 2=\{\mathrm{s} 3\}$
$\mathrm{C} 3=\{\mathrm{s} 4\}$
$\mathrm{C} 4=\{\mathrm{s} 1\}$
B. $D=\{s 5\}$
$\mathrm{C} 1=\{\mathrm{s} 2\}$
$\mathrm{C} 2=\{\mathrm{s} 3\}$
$\mathrm{C} 3=\{\mathrm{s} 4\}$
$\mathrm{C} 4=\{\mathrm{s} 1\}$
C. $\mathrm{D}=\{\mathrm{s} 1\}$
$\mathrm{C} 1=\{\mathrm{s} 2\} \quad \mathrm{C} 2=\{\mathrm{s} 3\}$
$\mathrm{C} 3=\{\mathrm{s} 4\}$
$\mathrm{C} 4=\{\mathrm{s} 5\}$
D. $\mathrm{D}=\{\mathrm{s} 1\}$
$\mathrm{C} 1=\{\mathrm{s} 2\}$
$\mathrm{C} 2=\{\mathrm{s} 3\}$
$\mathrm{C} 3=\{\mathrm{s} 4\}$
$\mathrm{C} 4=\{\mathrm{s} 5\}$
E. NOTA
110. Three players want to divide a cake fairly using the lone divider method. The divider cuts the cake into 3 slices. Describe a fair division given the following bids: $\mathrm{C} 1=\{\mathrm{s} 3\}, \mathrm{C} 2=\{\mathrm{s} 3\}$
A. $D=\{s 3\}$
$\mathrm{C} 1=\{\mathrm{s} 1\}$
$\mathrm{C} 2=\{\mathrm{s} 2\}$
B. $D=\{s 3\}$
$\mathrm{C} 1=\{\mathrm{s} 2\}$
$\mathrm{C} 2=\{\mathrm{s} 1\}$
C. $\mathrm{D}=\{\mathrm{s} 1\}$
$\mathrm{C} 1=\{\mathrm{s} 2\}$
$\mathrm{C} 2=\{\mathrm{s} 3\}$
D. $D=\{s 1\}$
C1 $=\{$ splits S2 and S3 with C2 $\}$
$\mathrm{C} 2=\{$ splits S2 and S3 with C1 $\}$

Consider the following problem for questions 111-115.
Justin and Jared are dividing a jeep using the Knaster Inheritance Procedure.

|  | Justin | Jared |
| :---: | :---: | :---: |
| bid | $\$ 10,000$ | $\$ 8000$ |
| fair share |  |  |
| amount received |  |  |
| amount owed |  |  |
| surplus |  |  |
| end result |  |  |
|  |  |  |

111. What is Justin's fair share?
A. $\$ 5000$
B. $\$ 3333.33$
C. $\$ 10,000$
D. NOTA
112. What is the amount that Justin received?
A. $\$ 5000$
B. $\$ 3333.33$
C. $\$ 10,000$
D. NOTA
113. What is the amount owed to Justin?
A. $\$-5000$
B. $\$-3333.33$
C. $\$-10,000$
D. NOTA
114. What is Justin's surplus?
A. $\$ 500$
B. $\$ 333.33$
C. $\$ 1000$
D. NOTA
115. What is Justin's end result?
A. Gets Jeep and $\$ 500$.
B. Gets Jeep and $\$ 4500$
C. Get Jeep and pays $\$ 500$.
D. Gets Jeep and pays $\$ 4500$.
E. NOTA.

Use the method of markers to divide the items. Then answer questions 116-121.

116. A gets:
A. $1,2,3$
B. 5,6
C. $8,9,10$
D. 12
E. 15,16
F. $4,7,11,13,14$
G. NOTA
117. B gets:
A. $1,2,3$
B. 5,6
C. $8,9,10$
D. 12
E. 15,16
F. $4,7,11,13,14$
G. NOTA
118. C gets:
A. $1,2,3$
B. 5,6
C. $8,9,10$
D. 12
E. 15,16
F. $4,7,11,13,14$
G. NOTA

119: D gets:
A. $1,2,3$
B. 5,6
C. $8,9,10$
D. 12
E. 15,16
F. $4,7,11,13,14$
G. NOTA
120. E gets:
A. $1,2,3$
B. 5,6
C. $8,9,10$
D. 12
E. 15,16
F. $4,7,11,13,14$
G. NOTA

121: The leftovers are:
A. $1,2,3$
B. 5,6
C. $8,9,10$
D. 12
E. 15,16
F. $4,7,11,13,14$
G. NOTA

Consider the problem for questions 122-130.
6 people ( $\mathrm{P} 1, \mathrm{P} 2, \ldots \mathrm{P} 6$ ) are dividing a cake using the Last Diminisher Method.
Each round has the following diminishers:
R1: P4, P5, P6
R2: No-one
R3: P3
R4: Anyone that can diminish does.
122. Who cuts the piece in round 1 ?
A. P1
B. P2
C. P3
D. P4
E. P5
F. P6
123. Who gets the piece in round 1 ?
A. P1
B. P2
C. P3
D. P4
E. P5
F. P6
124. Who cuts the piece in round 2 ?
A. P1
B. P2
C. P3
D. P4
E. P5
F. P6
125. Who gets the piece in round 2 ?
A. P1
B. P2
C. P3
D. P4
E. P5
F. P6
126. Who cuts the piece in round 3 ?
A. P1
B. P2
C. P3
D. P4
E. P5
F. P6
127. Who gets the piece in round 3 ?
A. P1
B. P2
C. P3
D. P4
E. P5
F. P6
128. Who is the last person with the opportunity to diminish the piece in round 4 ?
A. P1
B. P2
C. P3
D. P4
E. P5
F. P6
129. Who gets the piece in round 4 ?
A. P1
B. P2
C. P3
D. P4
E. P5
F. P6
130. How many rounds does it take to divid the cake among 6 people?
A. 1
B. 2
C. 3
D. 4
E. 5
F. 6
131. What is $14 \%$ of 68 ?
A. 485
B. 9.52
C. 0.18
D. 0.19
E. NOTA
132. 50.84 is what percent of 82 ?
A. $82 \%$
B. $72 \%$
C. $62 \%$
D. $1.6 \%$
E. NOTA
133. 75.14 is $34 \%$ of what number?
A. 25.5
B. 221
C. 134
D. 219
E. NOTA

Consider the problem to answer questions 131-137.
There are 50 TA 's to be apportioned to the following classes.

| Class | Population | Standard <br> Quota | Lower <br> Quota | Surplus <br> Apportionment |
| :---: | :---: | :---: | :---: | :---: |
| Algebra | 124 |  |  |  |
| Geometry | 135 |  |  |  |
| Trig | 152 |  |  |  |
| Senior Elective | 89 |  |  |  |
| Total | 500 |  |  |  |

134. What is the standard divisor?
A. 50
B. 500
C. 10
D. 11
E. NOTA
135. Fill in the standard quota for each class. (Hint: Divide the population by the standard divisor.) Fill in the lower quotas for each class. (Hint: Round down.) What is the total sum of the lower quota's?
A. 50
B. 49
C. 48
D. 47
E. NOTA
136. How many surplus seats are there?
A. 1
B. 2
C. 3
D. 4
E. NOTA
137. Using Hamilton's Method, how many TA's does Algebra get?
A. 9
B. 10
C. 11
D. 12
E. 13
F. 14
G. 15
H. NOTA
138. Using Hamilton's Method, how many TA's does Geometry get?
A. 9
B. 10
C. 11
D. 12
E. 13
F. 14
G. 15
H. NOTA
139. Using Hamilton's Method, how many TA's does Trig get?
A. 9
B. 10
C. 11
D. 12
E. 13
F. 14
G. 15
H. NOTA
140. Using Hamilton's Method, how many TA's does Senior Elective get?
A. 9
B. 10
C. 11
D. 12
E. 13
F. 14
G. 15
H. NOTA

Consider the problem for questions 138-153.
There are 50 nurses to be apportioned to 4 departments in a hospital with 730 patients.

| Department | Popula <br> tion | SQ <br> $\mathbf{d =}$ | Hamilto <br> $\mathbf{n ' s}$ | Jefferso <br> $\mathbf{n ' s}$ <br> $\mathbf{d =}$ | Adam's <br> $\mathbf{d =}$ | Webster' <br> $\mathbf{s}$ <br> $\mathbf{d}=$ | H-H's <br> $\mathbf{d}=$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Intensive <br> Care | 79 |  |  |  |  | - |  |
| Cardiac | 121 |  |  |  |  |  |  |
| Maternity | 233 |  |  |  |  |  |  |
| Pediatrics | 297 |  |  |  |  |  |  |
| Total | 730 | 50 | 50 | 50 | 50 | 50 | 50 |

141. Using Jefferson's Method, how many nurses does Intensive Care get?
A. 5
B. 6
C. 8
D. 9
E. 16
F. 20
G. 21
H. NOTA
142. Using Jefferson's Method, how many nurses does Cardiac get?
A. 5
B. 6
C. 8
D. 9
E. 16
F. 20
G. 21
H. NOTA
143. Using Jefferson's Method, how many nurses does Maternity get?
A. 5
B. 6
C. 8
D. 9
E. 16
F. 20
G. 21
H. NOTA
144. Using Jefferson's Method, how many nurses dies Pediatrics get?
A. 5
B. 6
C. 8
D. 9
E. 16
F. 20
G. 21
H. NOTA
145. Using Adam's Method, how many nurses does Intensive Care get?
A. 5
B. 6
C. 8
D. 9
E. 16
F. 20
G. 21
H. NOTA
146. Using Adam's Method, how many nurses does Cardiac get?
A. 5
B. 6
C. 8
D. 9
E. 16
F. 20
G. 21
H. NOTA
147. Using Adam's Method, how many nurses does Maternity get?
A. 5
B. 6
C. 8
D. 9
E. 16
F. 20
G. 21
H. NOTA
148. Using Adam's Method, how many nurses dies Pediatrics get?
A. 5
B. 6
C. 8
D. 9
E. 16
F. 20
G. 21
H. NOTA
149. Using Webster's Method, how many nurses does Intensive Care get?
A. 5
B. 6
C. 8
D. 9
E. 16
F. 20
G. 21
H. NOTA
150. Using Webster's Method, how many nurses does Cardiac get?
A. 5
B. 6
C. 8
D. 9
E. 16
F. 20
G. 21
H. NOTA
151. Using Webster's Method, how many nurses does Maternity get?
A. 5
B. 6
C. 8
D. 9
E. 16
F. 20
G. 21
H. NOTA
152. Using Webster's Method, how many nurses dies Pediatrics get?
A. 5
B. 6
C. 8
D. 9
E. 16
F. 20
G. 21
H. NOTA
153. Using Huntington-Hill's Method, how many nurses does Intensive Care get?
A. 5
B. 6
C. 8
D. 9
E. 16
F. 20
G. 21
H. NOTA
154. Using Huntington-Hill's Method, how many nurses does Cardiac get?
A. 5
B. 6
C. 8
D. 9
E. 16
F. 20
G. 21
H. NOTA
155. Using Huntington-Hill's Method, how many nurses does Maternity get?
A. 5
B. 6
C. 8
D. 9
E. 16
F. 20
G. 21
H. NOTA
156. Using Huntington-Hill's Method, how many nurses dies Pediatrics get?
A. 5
B. 6
C. 8
D. 9
E. 16
F. 20
G. 21
H. NOTA
157. Round the quota 12.4213 using Huntington-Hill's method.
A. 12
B. 13
C. NOTA
